SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report of 2022: SVP-2229

"Black Scholes Model and its Applications"

IQAC
Sri Venkateswara College
University of Delhi
Dhaula Kuan
New Delhi - 110021

SRIVIPRA PROJECT 2022

Title : Black Scholes Model and its Applications

<table>
<tr><td>Name of the mentor :Dr Rajni Arora
Name of Department: Mathematics
Designation: Assistant Professor</td><td></td></tr>
</table>

<table>
<tr><td>Name of the mentor :Dr Garima Virmani Arora
Name of Department: Mathematics
Designation: Assistant Professor</td><td></td></tr>
</table>

List of students under the SRIVIPRA Project

S. No.	Name of the student	Course	Photo
1	**Palak Chaudhary**	**B.Sc.(Hons.) Mathematics**	
2	**Sameer Gupta**	**B.Sc.(Hons.) Mathematics**	
3	**Ayush Gupta**	**B. Sc. (Hons.) Statistics**	

4	**Arushi Gandhi**	**B.Sc. (Hons.) Mathematics**	
5	**Jahnvi Agarwal**	**B.Sc. (Hons.) Mathematics**	

Signature of Mentors

Sri Venkateswara College

University of Delhi

SRIVIPRA-2022

(Sri Venkateswara College Internship Program in Research and Academics)

This is to certify that this project on Black Scholes Model and its Applications was registered under SRIVIPRA and completed under the mentorship of ~~Prof.~~/Dr./ ~~Mr.~~/Ms. Rajni Arora and Dr Garima Virmani Arora during the period from 21st June to 7th October 2022.

Sharda Pasricha and S. Krishnakumar

Coordinators

Prof. C Sheela Reddy

Principal

CERTIFICATE

This is to certify that the aforementioned students from Sri Venkateswara College participated in the summer project SVP-2229 titled "**Black Scholes Model and its Applications**". The participants have carried out the research project work under our guidance and supervision from 21st June 2022 to 7th October 2022. The work carried out is original and carried out in online/ offline mode.

Signature of Mentors

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CONTENTS

BLACK SCHOLES MODEL AND ITS APPLICATIONS

Abstract

In the current world, the continuous evolution of the financial market has led to the development of unique financial instruments. One such class of instruments is known as the financial derivatives. The financial derivatives are the type of instruments that derive their value from an underlying asset, group of assets or benchmarks with their prices being determined and affected by the fluctuations in the underlying asset. One such financial instrument is Option which is a financial contract between two or more parties where one party (buyer of the contract) has a right or an obligation to purchase (or sell) the underlying asset if the option is call (or put) based. Since the options are so dependent on the prices of the underlying asset, the changes in the underlying assets lead to a subsequent change in the option prices.

Thus, option pricing is one of the most popular and complex areas in finance which has been extensively developed since the 1970's after the introduction of Black-Scholes Model. In India, option trading was formalized in the mid-2000's.

This study is to understand and determine the concepts on which the Black-Scholes Model was made by discussing the derivation and formula of the model along with testing the theory that it depends on, which refers to the factors which affect the option pricing, which had been experienced and studied over the years. It will also consider the relevance of Black Scholes Model in the Indian Market as it compares the Black Scholes Model Option Pricing (BSMOP) and the actual pricing of underlying index for the NIFTY (Bank NIFTY) in the case of call (put) option.

1. Introduction/Background

1.1 DERIVATIVES

Derivatives are financial contracts between two or more parties and whose value is derived from an underlying asset, group of assets, or benchmark. A derivative can be traded on an exchange or off-exchange. Derivative prices are determined by fluctuations in the underlying asset.

Types of derivatives

Futures

Futures are derivative financial contracts that obligate parties to buy or sell an asset at a predetermined future date and price. The buyer must purchase or the seller must sell the underlying asset at the set price, regardless of the current market price at the expiration date.

Forwards

A forward contract is an obligation to buy or sell a certain asset at a specified price (forward price) and at a specified time (contract maturity or expiration date). These are typically not traded on exchanges.

Swaps

A swap Derivative is a contract wherein two parties decide to exchange liabilities or cash flows from separate financial instruments. Often, swap trading is based on loans or bonds, otherwise known as a notional principal amount. However, the underlying instrument used in Swaps can be anything as long as it has a legal, financial value. Mostly, in a swap contract, the principal amount does not change hands and stays with the original owner. While one cash flow may be fixed, the other remains variable and is based on a floating currency exchange rate, benchmark interest rate, or index rate.

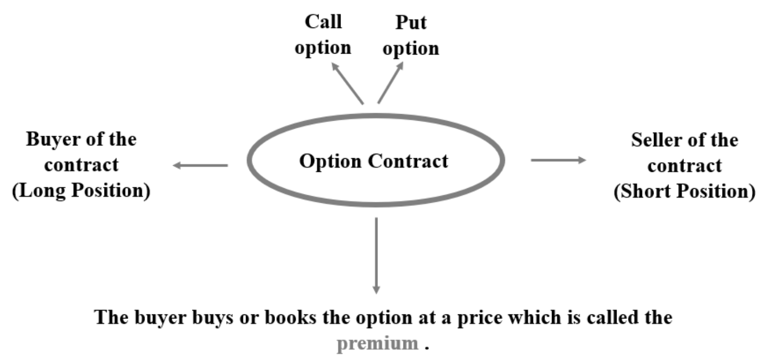
1.2 OPTIONS

Options are derivatives that, unlike futures, are not binding, that is, you have the right but not an obligation to buy or sell the option on a stipulated date (**expiration date**) at a fixed price (**strike price**). To simply put it, there is an "option" to exercise the contract. Before diving deeper into the options, let us first take a look at an example.

Mr. X is thinking of buying a car that is to launch in the market next month. One day, a friend of his, working in a GST office, informs him that there could be a plausible hike in rates of GST from the next month. So, Mr. X books the car at a premium price of Rs. 10,000 under the terms that even if the GST rises, he will be sold the car at present rate of GST. Now, the next month instead of increasing the GST rate falls and Mr. X decides not to exercise the contract he got made.

So, in the above example Mr. X was not obligated to buy the car under any circumstances and it was totally up to his discretion whether he wanted to exercise the contract or not.

The initial price of Rs. 10.000 at which Mr. X books an option of that car is called **premium**. While trading in options, loss is always equal to the premium price only.



TYPES OF OPTIONS

<u>Call option</u> is the right given to the owner of the option to buy an underlying asset which includes a stock, bond or foreign currency within a given period from the writer (seller).

- **Strike price < Market Price**

Type of Option Contract – In The Money (ITM) (it basically represents the profit opportunity the buyer will have as the market price > strike price and they can exercise the option at lower strike price and can sell at higher market price to secure a profit.). It possesses intrinsic value.

- **Strike price > Market Price: -** as the options have an expiration date, the call expire unused and worthless

Type of Option Contract – Out The Money (OTM) (it has no intrinsic value, only extrinsic value. In this case, if the trader had bought the option with more extrinsic value i.e. more time will remain for the option to expire which will imply that the option will possess intrinsic value i.e. it will move closer to being in the money and as stated above, the profit can be secured.)

- **Strike price = Market Price**

Type of Option Contract – At The Money (ATM) (these are sensitive to changes in risk factors as these options are very close to having intrinsic value.)

Where Strike price = stated price on an option.

Put option is opposite of call option i.e. it holds the right to the owner to sell the underlying asset.

- **Strike price > Market Price**

Type of Option Contract – In The Money (this means that the put option holder has the right to sell the asset at a price which is greater than the market price, hence secures a profit.)

- **Strike price < Market Price: -** as the options have an expiration date, the call expire unused and worthless

Type of Option Contract – Out The Money (since it has no intrinsic value, only extrinsic value so, in this case, if the trader had bought the option with more extrinsic value i.e. more time will remain for the option to expire which will imply that the option will possess intrinsic value i.e. it will move closer to being in the money and as stated above, the profit can be secured.)

- **Strike price = Market Price**

Type of Option Contract – At The Money (these are sensitive to changes in risk factors as these options are very close to having intrinsic value.)

Call options and put options are subjected to increase or decrease by the Derivatives traders which amounts the risk that they take.

Intrinsic Value

It is the measure of defining the worth of an asset, whose formula is given by

Intrinsic Value = Strike price of the option – Market price of the option

Extrinsic Value

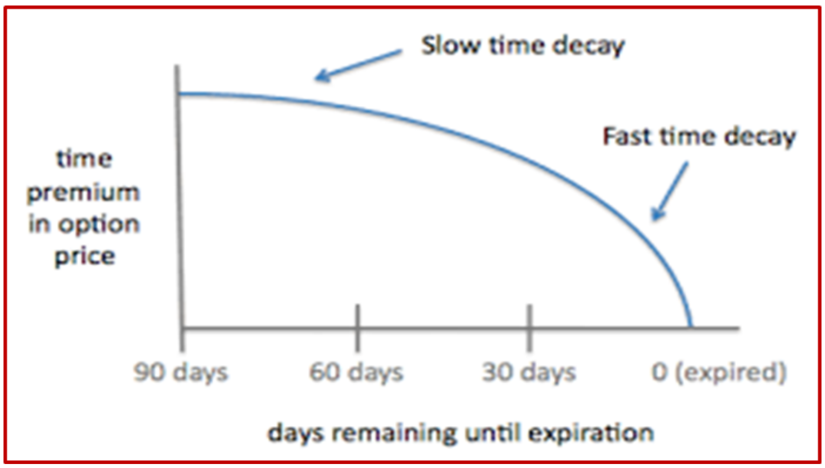
It is defined as the time remaining for the option to retain its monetary value until the expiration date arrives or until the option expires (as every option is a contract consisting of a specific expiration period.)

It is also referred to as Time Value.

Time Decay

As the option approaches the expiration date, the monetary value will be associated with the option for a lesser period of time i.e. the extrinsic value will decrease and hence less time will be available to the holder to earn profit. This process of declining extrinsic value is known as Time Decay.

When the expiration date draws near, extrinsic value decreases at an accelerating pace and eventually reaches zero. Hence, both time value and time decay play vital roles in determining how much profit opportunities are available to the option holder.



From the above understanding, we can say that the options with more intrinsic value are more in sync with the stock price (ITM/ATM), whereas the options with extrinsic value are less sensitive to the stock's price movement and more in sync with the market price (OTM).

Option Pricing

Option pricing is a way of estimating the monetary value(fair value) of an options contract by assigning a specific price, known as premium, which traders incorporate into their strategies.

FACTORS AFFECTING OPTION PRICING

Interest Rates

Interest rates affect the whole economy in which they prevail. Interest rate changes also affect the option pricing. A **rise in interest rate changes leads to increase in call option prices and decrease in put option prices**.

We can explain it as: For **Call Options**, the situation can be that a person wants to buy 1000 units of shares with a price of Rs. 100 or Rs 100000 to spend/borrow. They can spend Rs. 6 on a call option that gives them the right to buy the share units and spend Rs. 6000 and get the same benefits as they can buy a share unit (assuming that they believe they will buy it because they expect the share price to rise). Hence, by buying call options, they save Rs. 94000. They may not need to borrow the huge amount of money at the interest rate that prevails. This means that at a higher interest rate, the opportunity cost of switching to buying options will be less and hence, it will be more favorable to buy the options. The saved money can be saved in the bank and hence, at a higher interest rate, you will get more benefit from that money. For example, If interest rate is 5%, we save 94000*0.05= Rs. 4700. This makes the options more valuable.

For **Put Options**, the situation can be that the person wants to sell 1000 units of shares with a price of Rs. 100 or Rs 100000 worth of units to sell. Otherwise, they can spend Rs. 6 on a put option that gives them the right to sell the units for Rs. 6000 and can get the same benefits as they can by selling the units now (assuming they believe they will sell it because they expect the share price to fall). If the person chooses to sell now, they can earn Rs 100000 currently and also save themselves from spending Rs 6000 on buying the put options. If the current interest rate is 5%, they can earn about 106000*0.05=Rs. 5030 by saving it in the bank. Hence, at a higher interest rate, if the investor were to sell the units rather than buy put options, it will be more favorable and this is why the value of put options decreases for the investor at higher interest rates.

Models such as Black Scholes Model use the risk-free interest rates such as annualized interest rates from the bonds issued by Govt. or MIBOR. These are considered as risk-free since they are the most secured bonds or reflect the minimum rate at which no risk is faced by the investor.

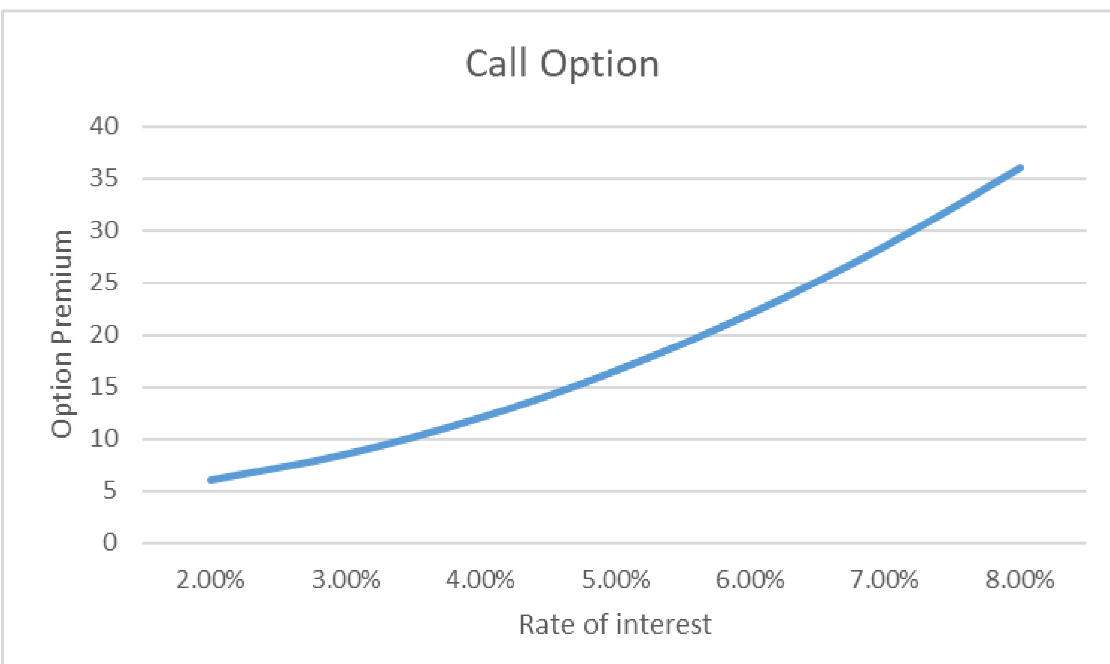


Figure 1 The Rate of Interest vs. Option Premium for Call Option

As the rate of interest increases, the option premium for a call option is expected to increase.

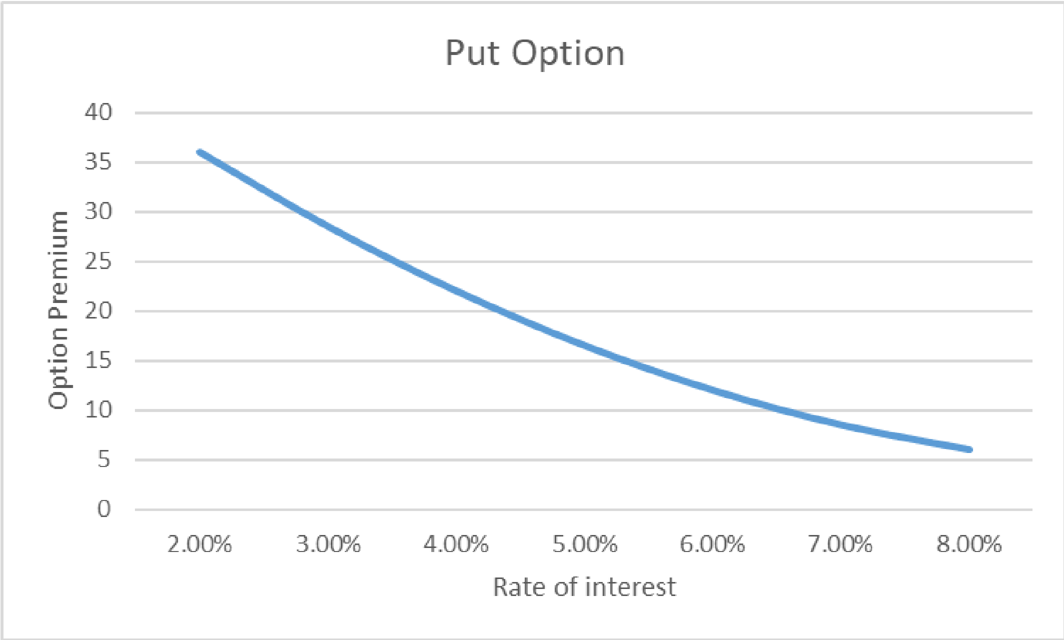


Figure 2 The Rate of Interest vs. Option Premium for Put Option

As the rate of interest increases, the option premium for a put option is expected to decrease.

Time till Expiration

The options may have two values: intrinsic and extrinsic values where intrinsic value is the value of option when it was exercised immediately whereas the time to expiration gives extrinsic value.

The time value of the option increases as there is more time until the expiration of the contract as intuitively, it suggests the gains that the option could possibly have by change in the prices of stock in future.

For example, let it be January and let us assume a call/put option with a strike price of Rs. 100 as well as current spot price of Rs. 100. An option that may be expiring in 1 month (in February) may be valued at Rs. 4.9, the option expiring in 2 months (in March) may be valued at Rs. 8 and the option expiring in 3 months may be valued at Rs. 10. The option premium increases because the investor expects more time for changes in price of stock to assume a gain in future.

At the time of expiration, there may not be any extrinsic value left so if the above option were to expire at the date of purchase, the value of the option will be Rs. 0 (no extrinsic value).

The options with different stock prices and different time periods may be expressed as following:

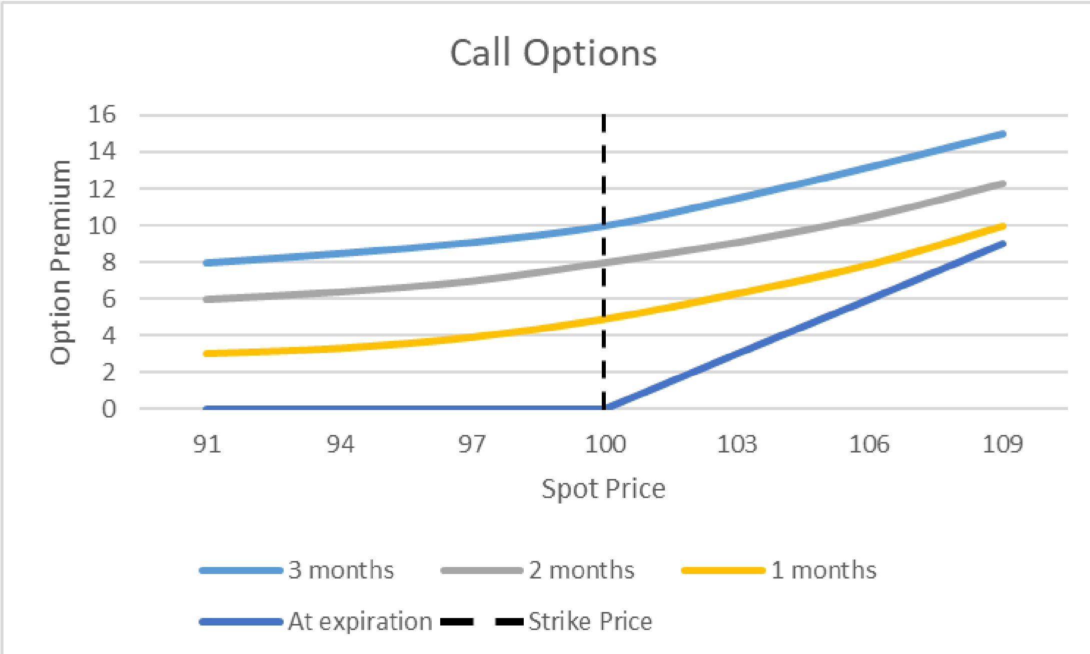


Figure 3 The Option Premium of a Call Option v.s. The Spot price of the Options for different times to expiration

The option premium increases with an increase in the stock current (spot) prices. And as the time till expiration is more, the option premium for the call options is more.

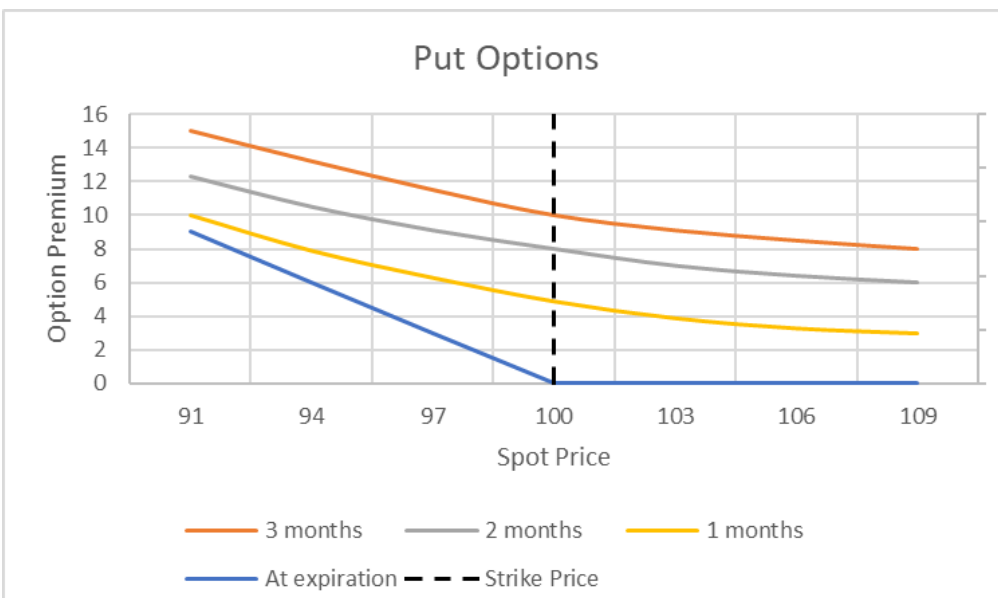


Figure 4 The Option Premium of a Put Option v.s. The Spot price of the Options for different times to expiration

The option premium for put options increases with a decrease in the stock current (spot) prices. And as the time till expiration is more, the option premium for the put options is more.

Volatility

Volatility refers to the variance in the stock market leading to deviations in prices of stock. There may be two types of volatility: historical (calculated by previous prices of the stock market) and implied (predicted volatility of market in future for the span of the option duration time)

Historical volatility can be calculated using the historical returns observed in the market whereas implied volatility may be calculated using formulas measuring option market expectations

The higher the volatility, the more will be the value of the options for both call and put options as there will be more fluctuations leading to unexpected valuation in the future.

Hence, if for a call/put option of premium of Rs. 10 and the volatility were to increase from 12% to 18% pa, the value of the same call/put option will increase and it may be Rs. 16.

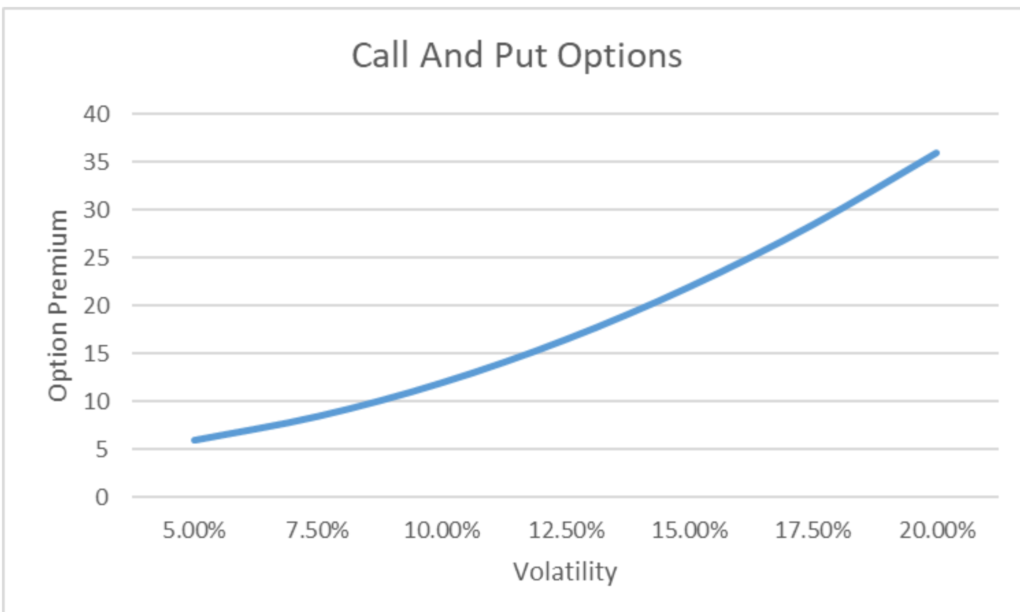


Figure 5 Option Premium of a Call/Put Option v.s. The Volatility of the Stock prices.

The option premium for any option, regardless of it being call or put, will have an increase in its value as the volatility of the stock prices is increased.

Stock Price

The stock price or underlying price refers to the current price at which the underlying is being traded in the market.

For call options, if the stock price goes up, their intrinsic value increases as they will have the right to buy the asset at a price lower than the stock price.

For put options, if the stock price goes up, their intrinsic value decreases as they will have the right to sell the asset at a price lower than the current stock price.

The relation of stock price with the option's valuation can be determined in the graph and the example it uses.

Strike Price

It refers to the predetermined price at which the buyer of the option can exercise their right to buy or sell the underlying asset.

Its effect is in the form of intrinsic value it provides in the option. More in-the-money the option is, the more intrinsic value the option will have.

It can be dictated by strike price as: the less the strike price is to the current stock price for a call option, the more intrinsic value it has. For put options, the more the strike price is to the current stock price, the more intrinsic value it has.

For call option, if an option X has strike price of Rs. 90 and other option Y has strike price of Rs. 80 with the option having same stock with price of Rs. 100, then X will have intrinsic value of Rs. 10 whereas Y will have intrinsic value of Rs. 20 and hence, more value.

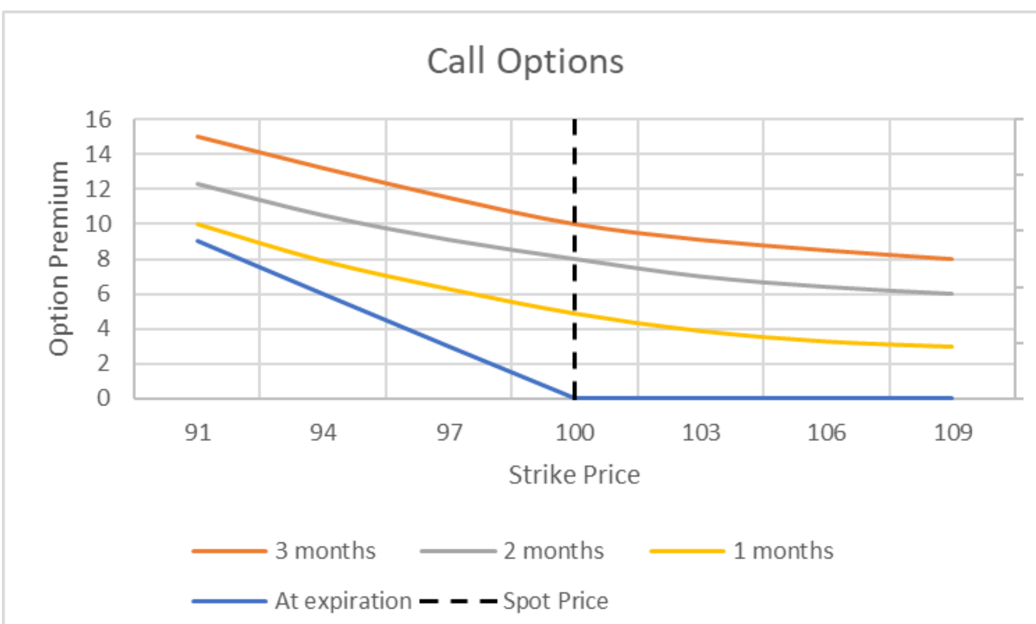


Figure 6 The Option Premium of a Call Option v.s. The Strike price of the Options for different times to expiration

The option premium for a call option increases as the strike price for the option decreases, and as is known, as the time increases, the value of the options increases.

Figure 7 The Option Premium of a Put Option v.s. The Strike price of the Options for different times to expiration

The option premium of a put option increases as the strike price increases, and as is known, the value of option is higher when the time till expiration is more.

1.3 BLACK SCHOLES MODEL

History of the Black Scholes Model

The Black-Scholes model, developed in 1973 by Fischer Black, Robert Merton, and Myron Scholes, was the first widely used mathematical method for calculating the theoretical value of an option contract using current stock prices, expected dividends, the option's strike price, expected interest rates, time to expiration, and expected volatility.The original equation was introduced in Black and Scholes' 1973 paper, "The Pricing of Options and Corporate Liabilities," published in the Journal of Political Economy, introduced the initial equation. Robert C. Merton assisted in the editing of that paper. Later that year, in The Bell Journal of Economics and Management Science, he published his own article, "Theory of Rational Option Pricing," expanding the mathematical understanding and applications of the model and coining the term "Black-Scholes theory of options pricing."

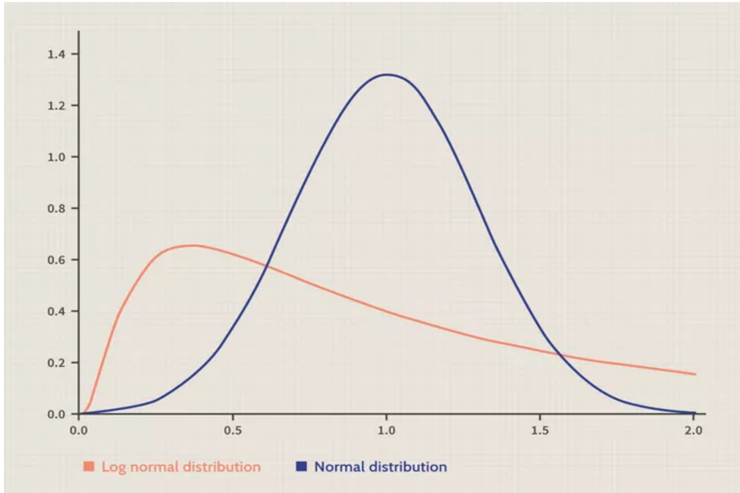
Scholes and Merton received the Nobel Memorial Prize in Economic Sciences in 1997 for developing "a new method to determine the value of derivatives." Black had died two years before and thus could not be a recipient because Nobel Prizes are not awarded posthumously; however, the Nobel committee recognised his role in the Black-Scholes model.

What is Black Scholes Model ?

The Black-Scholes-Merton (BSM) model, one of the most important concepts in modern financial theory, is a pricing model for financial instruments. It is used to determine the value of stock options. This model is used to calculate the fair value of stock options by taking into account five variables: volatility, underlying stock price, strike price, time, and risk-free rate. It is based on the hedging principle and aims to eliminate risks associated with the volatility of underlying assets and stock options. The standard Black Scholes Model is only used to price European options because it does not account for the possibility of exercising American options before the expiration date.

Lognormal Distribution of Stock Prices

Normal distribution is the probability distribution of outcomes that is symmetrical or forms a bell curve. 68% of the result falls within one standard deviation and 95% of the result falls into two standard deviations. A lognormal distribution has a lower bound of zero and is skewed to the right, i.e., it has a long right tail. This is in contrast with a normal distribution which has zero skew and can take both negative and positive values.



Lognormal distribution plots the logarithm of a random variable from a normal distribution curve. When the natural logarithm of a random variable is normally distributed, then the variable itself will have a lognormal distribution. Stock prices changes are random and therefore it follows a normal curve but stock prices themselves follow a lognormal distribution.

The model of stock price behavior used by Black, Scholes, and Merton assumes that percentage changes in the stock price in a very short period of time are normally distributed. Define :

$$\mu : \textit{Expected return on stock per year}$$

$$\sigma : \textit{Volatility of the stock price per year}$$

The mean and standard deviation of the return in time Δt are approximately $\mu\Delta t$ and $\sigma\sqrt{\Delta t}$, so that

$$\frac{\Delta S}{S} \sim \phi(\mu\Delta t , \sigma^2 \Delta t)$$

where ΔS is the change in the stock price S in time Δt, and $\phi(m, v)$ denotes a normal distribution with mean m and variance v.

Stochastic Processes

The question that might pop in your head is why are we studying stochastic processes in Black Scholes?
The reason being that Brownian Motion is also a type of stochastic process and Brownian motion plays a vital role in Black Scholes. So, before diving into Brownian Motion, we first need to get a brief about stochastic processes and the Markov Model.

The word stochastic means “random” and is concerned with change in time. Stochastic processes are processes that govern how random variables change over time. These are heavily used for the analysis and forecasting process.

Example - If we have to predict the probability of a student reaching late to school the next day, we will have to see whether he was on time today or not.

Notation - $X_n = i$

where, X = the Process or random variable

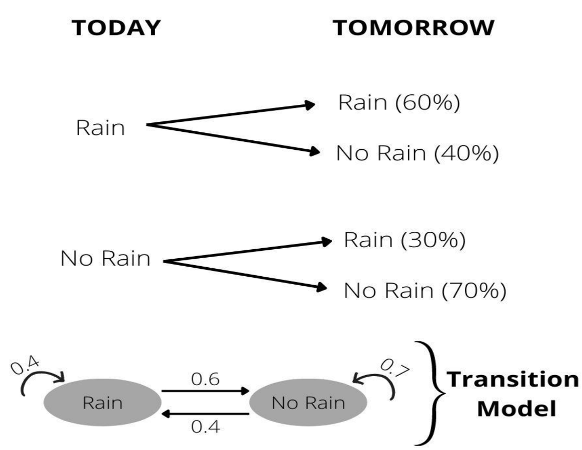
n = time period

i = state of the variable/process

MARKOV MODEL

A Markov model is a stochastic model that is used to model systems that change randomly over time. It is assumed that future states are determined solely by the current state and not by previous events. It mainly depends on two things: state of the variable and Transition Probability Matrix.

Example - The diagram below represents the Markov Transition model predicting the probability of rain



Here, in the above diagram, rain and no rain are the states of the variable.

BROWNIAN MOTION

Brownian Motion is nothing but a more continuous form of random walk process. It is a particular type of Markov stochastic process with a mean change of zero and a variance rate of 1.0 per year.

We say a variable z is following a Brownian motion if it satisfies the following two properties:

Property 1. The change Δz during a small period of time Δt is

$$\Delta z = \varepsilon\sqrt{\Delta t}$$

where ε has a standard normal distribution.

Property 2. The values of Δz for any two different short intervals of time, Δt, are independent.

It follows from the first property that Δz itself has a normal distribution with

mean of Δz = 0

standard deviation of Δz = $\sqrt{\Delta t}$

variance of Δz = Δt

The second property implies that z follows a Markov process.

Consider the change in the value of z during a relatively long period of time, T. This can be denoted by z(T) - z(0). It can be regarded as the sum of the changes in z in N small time intervals of length t, where

$$N = \frac{T}{\Delta T}$$

Thus, from the above equation we can write

$$z(T) - z(0) = \sum_{i=1}^{N} \varepsilon_i\sqrt{\Delta t}$$

where ε_i (i = 1, 2, ... ,N) are distributed $\phi(0, 1)$. We know from the second property of Brownian motion is that the ε_i are independent of each other. Thus, $z(T) - z(0)$ is normally distributed, with,

mean of [z(T)-z(0)] = 0

variance of [z(T)-z(0)] = NΔt = T

standard deviation of [z(T)-z(0)] = $\sqrt{T}$

A more generalized and practical approach of Brownian Motion

The drift rate and variance rate for a stochastic process are the mean change and variance, respectively, per unit of time. The fundamental Brownian motion, dz, that has so far been established has drift and variance rates of 0 and 1, respectively. The expected value of z at any point in the future will be identical to its current value since the drift rate is zero. The variance of the change in z over a time interval of length T is equal to T when the variance rate is T, or 1.0. In terms of dz, a generalized Brownian motion can be characterized as

$$dx = a\,dt + b\,dz$$

where a and b are constants.

- *a dt* means that x has an expected drift rate of *a* per unit of time.

The equation is ***dx = a dt*** without the b dz term, which means that *dx/dt* = *a*. Taking into account time integration, we obtain

$$x = at + x0$$

where x0 is the value of x at time 0. In a period of time of length T, the variable x increases by an amount *aT*. The *b dz* term variability to the path followed by *x*. The amount of this variability is b times a brownian motion. A brownian motion has a variance rate per unit time of 1.0. It follows that b times a brownian motion has a variance rate per unit time of b^2. In a small time interval Δt, the change Δx in the value of x is given by equation,

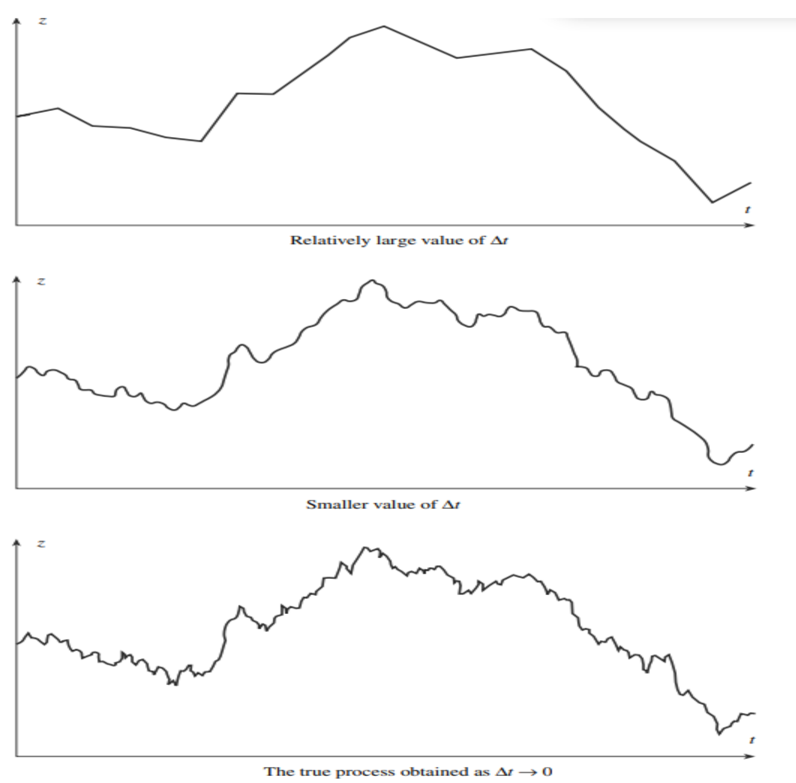
$$\Delta x = a\Delta t + b\varepsilon\sqrt{\Delta t}$$

where, as before, ε has a standard normal distribution ϕ(0,1). Thus Δx has a normal distribution with

$$\text{mean of } \Delta x = a\Delta t$$

$$\text{standard deviation of } \Delta x = b\sqrt{\Delta t}$$

$$\text{variance of } \Delta x = b^2\Delta t$$



ITO's PROCESS

An n-dimensional Ito process is an example of a stochastic differential equation where X_t evolves like a Brownian motion with drift $a(t, X_t)$ and standard deviation $b(t, X_t)$. Moreover, we say that X_t is a solution to such a stochastic differential equation if it satisfies

$$X_t = X_0 + \int_0^t a(s, X_s)ds + \int_0^t b(s, X_s)dW_s$$

where, X_0 is a constant.

A more simplified way of writing the above equation is,

$$dx = a(x, t)dt + b(x, t)dz$$

Where variable x follows the Ito's process, dz is a Wiener process and a and b are functions of x and t.

Ito's lemma

We will use the above equation defined in Ito's process, that is,

$$dx = a(x, t)dt + b(x, t)dz \quad \ldots(1)$$

Ito's lemma shows that a function G of x and t follows the Ito process,

$$dG = (\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2)dt + \frac{\partial G}{\partial x}bdz$$

with a drift rate of

$$\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2$$

and a variance rate of

$$(\frac{\partial G}{\partial x})^2 b^2$$

In the assumptions of Black Scholes, we take the price of the underlying share such that it follows a geometric brownian motion

$$dS = \mu S dt + \sigma S dz$$

with μ and σ as constants.

So from Ito's process, both S and G are affected by the same underlying source of uncertainty, dz, that is,

$$dG = (\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2)dt + \frac{\partial G}{\partial S}\sigma S\, dz$$

ASSUMPTION OF BLACK SCHOLES MODEL

1) The price of the underlying share follows a geometric brownian motion
$$dS = \mu S dt + \sigma S dz$$
2) There are no risk free arbitrage opportunities.
3) The risk-free rate is constant, same for all maturities and some for borrowing and lending.

4) Unlimited short selling is allowed.
5) There are no taxes,costs and expenses.
6) The asset can be traded continuously and in small units.

DERIVATION OF BLACK SCHOLES MODEL

We consider a derivative's price at a general time t (not at time zero). If T is the maturity date, the time to maturity is T - t.

$$dS = \mu S dt + \sigma S dz \quad \ldots(1)$$

Suppose that f is the price of a call option or other derivative contingent on S. The variable f must be some function of S and t. Hence,

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right) dt + \frac{\partial f}{\partial S}\sigma S\, dz \quad \ldots(2)$$

The discrete versions of equations (1) and (2) are

$$\Delta S = \mu S \Delta t + \sigma S \Delta z \quad \ldots(3)$$

and

$$\Delta f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right) \Delta t + \frac{\partial f}{\partial S}\sigma S\, \Delta z \quad \ldots(4)$$

where Δf and ΔS are the changes in f and S in a small time interval t. From Itoˆ's lemma, the Brownian motion underlying f and S are the same. As a result, a portfolio of the stock and the derivative can be built to eliminate the Brownian motion.The portfolio is

-1: derivative

$+\ \partial f/\partial S$: shares

The holder of this portfolio is short one derivative and long an amount $\partial f/\partial S$ of shares. Define Π as the value of the portfolio. By definition

$$\Pi = -f + \frac{\partial f}{\partial S} S \quad \ldots(5)$$

The change in the value of the portfolio in the time interval t is given by

$$\Delta\Pi = -\Delta f + \frac{\partial f}{\partial S}\Delta S \quad \ldots(6)$$

Substituting equations (3) and (4) into equation (5) yields

$$\Delta\Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right) \Delta t \quad \ldots(7)$$

Because this equation does not involve z, the portfolio must be riskless during time t. The assumptions listed in the preceding section imply that the portfolio must instantaneously earn the same rate of return as other short-term risk-free securities. If it earned more than this return, arbitrageurs could make a riskless profit by borrowing money to buy the portfolio; if it earned less, they could make a riskless profit by shorting the portfolio and buying risk-free securities. It follows that

$$\Delta\Pi = r\Pi\Delta t \quad \ldots(8)$$

where r is the risk-free interest rate. Substituting from equations (5) and (6) into (8), we obtain

$$\left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(f - \frac{\partial f}{\partial S}S\right)\Delta t \quad \ldots(9)$$

so that

$$\frac{\partial f}{\partial t} + r\frac{\partial f}{\partial S}S + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 = rf$$

Equation (9) is the Black–Scholes–Merton differential equation. It has many solutions, corresponding to all the different derivatives that can be defined with S as the underlying variable. The particular derivative that is obtained when the equation is solved depends on the boundary conditions that are used. These specify the values of the derivative at the boundaries of possible values of S and t. In the case of a European call option, the key boundary condition is

$$f = max(S - K, 0) \quad when\, t = T$$

In the case of a European put option, it is

$$f = max(K - S, 0) \quad when\, t = T$$

2. Hypothesis/Goal of the Study

2.1 To determine the impact of change in a single factor affects the call and put option pricing in the Black Scholes Model

2.2 To Test Whether there is significant difference between Fair Price (price calculated by Black Scholes Model) and Actual Price.

3. Materials/Methods

3.1 Material and Research Methodology for testing whether there is significant difference between Fair Price (price calculated by Black Scholes Model) and Actual Price.

NIFTY Call options are taken from 1^{st} August,2022 to 31^{st} August 2022 expiring on 15^{th} September,2022.

Bank NIFTY Put options are taken from 18^{th} August,2022 to 12^{th} September,2022 expiring on 15^{th} September,2022.

Data for call Option

https://www.nseindia.com/get-quotes/derivatives?symbol=NIFTY&identifier=OPTIDXNIFTY15-09-2022CE18000.00

Data for current prices NIFTY

https://finance.yahoo.com/quote/%5ENSEI/history?period1=1659312000&period2=1661904000&interval=1d&filter=history&frequency=1d&includeAdjustedClose=true

Data for Put Option

https://www.nseindia.com/get-quotes/derivatives?symbol=NIFTY&identifier=OPTIDXNIFTY15-09-2022PE17900.00

Data for current prices Bank NIFTY

https://finance.yahoo.com/quote/%5ENSEBANK/history/

We have studied a sample each for Call and put options. We calculated Fair Prices using Black Scholes Model.

Paired T-test

It is used to compare the means of two samples.

H0 : = $\mu_1 = \mu_2$

H1 := $\mu_1 \neq \mu_2$

$$\text{Test statistic :- t} = \frac{\bar{x}_{diff}}{\frac{s_{diff}}{\sqrt{n}}}$$

- $\bar{x}_{diff}$: Sample mean of differences
- s_{diff} : Sample standard deviation of differences
- n : Sample size

To test whether there is significant difference between Actual Prices and Fair Prices, Paired T test is used.

HO (Null Hypothesis): There is no significant difference between Actual and Fair prices.

H1 (Alternate Hypothesis): There is a significant difference between Actual and Fair prices.

At 95% level of significance

If P-Value > 5%, We cannot be able to reject the null hypothesis.

Or if P-Value < 5%, We can reject null hypothesis.

4. Results/Key Findings

4.1 IMPACT OF CHANGE IN FACTORS ON OPTION PRICING

Call Option

Call Premium Option when the Rate of Interest is changed

It can be seen that as the risk-free rate of interest is increased, the call premium increases. This proves the theory of the impact of risk-free interest rate on the call option premium.

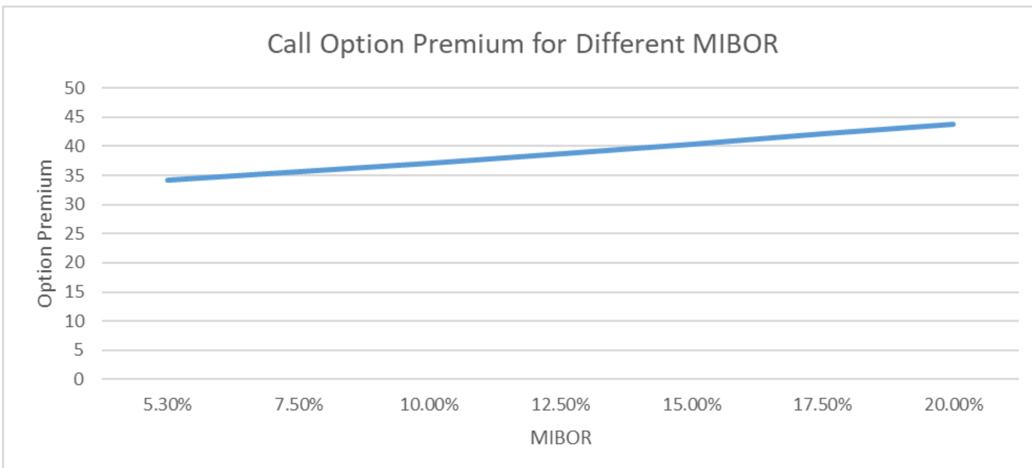


Figure 8 Change in Call Option Premium for different Rates of Interest

Call Premium Option when the Time till Expiration is changed

It can be seen that as the time till expiration is increased, the call premium increases. This proves the theory of the impact of time till expiration on the call option premium.

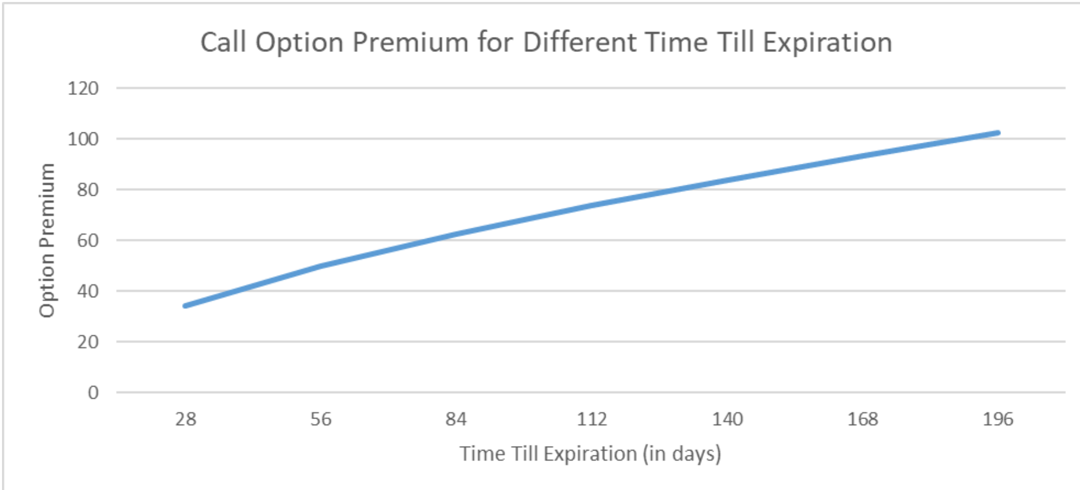


Figure 9 Change in Call Option Premium for different time till expiration

Call Premium Option when the Implied Volatility is changed

It can be seen that as the volatility is increased, the call premium increases. This proves the theory of the impact of volatility on the call option premium.

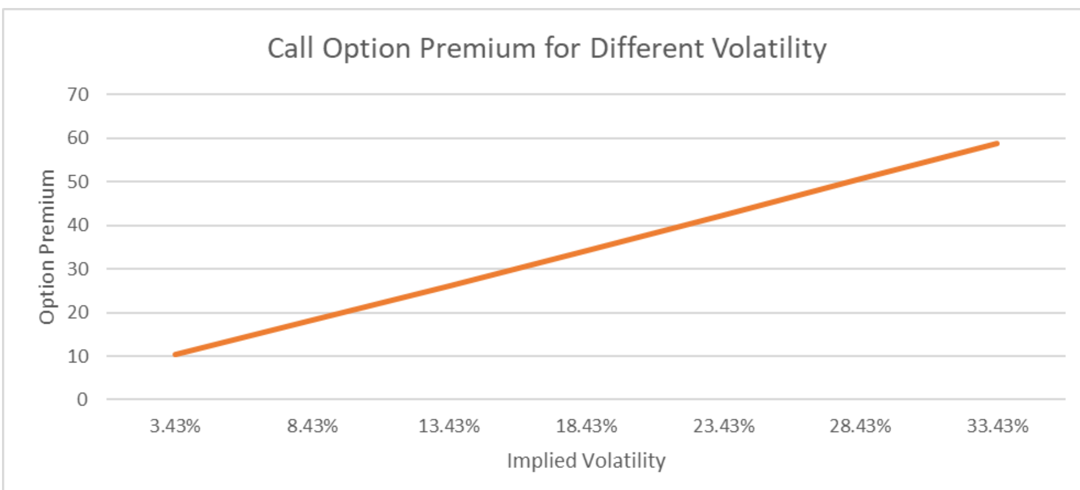


Figure 10 Change in Call Option Premium for different Volatility

Call Premium Option when the Spot Price is changed

It can be seen that as the spot price is increased, the call premium increases. This proves the theory of the impact of spot price on the call option premium.

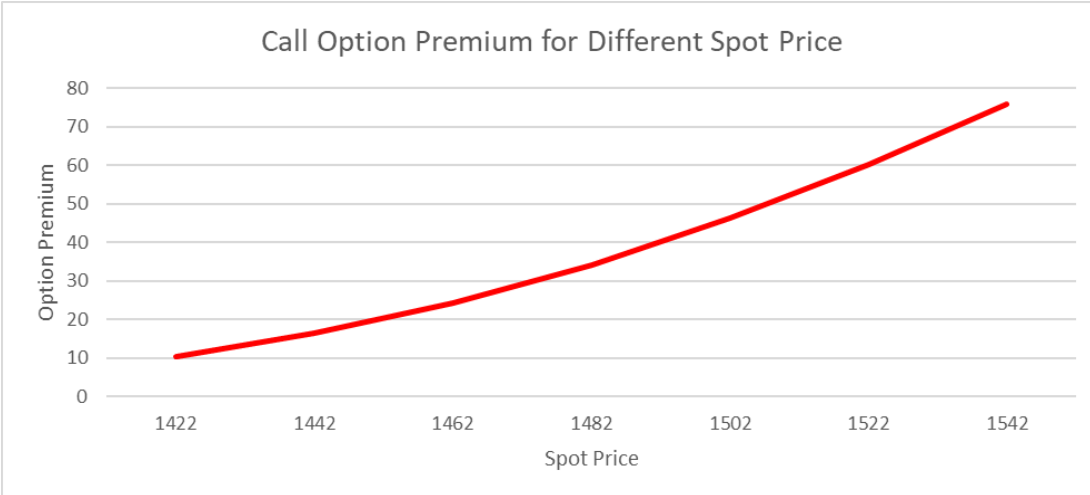


Figure 11 Change in Call Option Premium for different Spot Price

Call Premium Option when the Strike Price is changed

It can be seen that as the strike price is increased, the call premium decreases. This proves the theory of the impact of strike price on the call option premium.

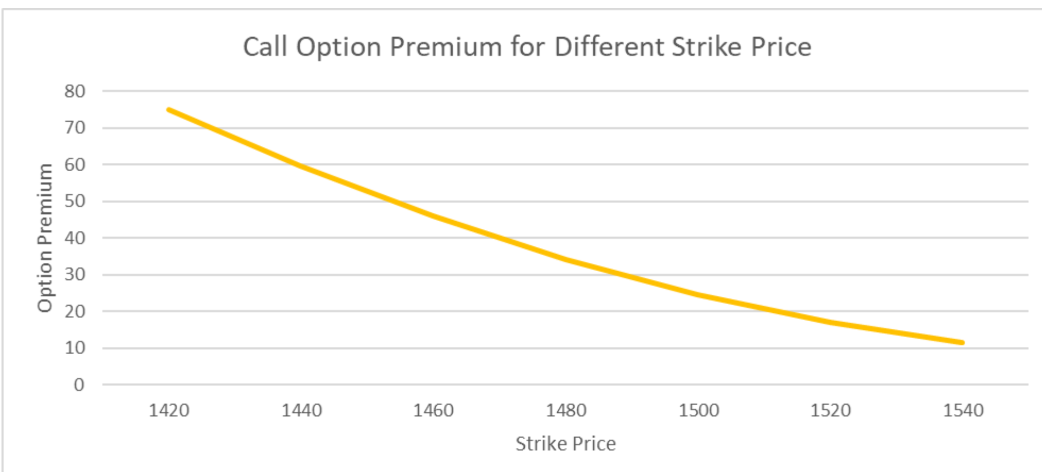


Figure 12 Change in Call Option Premium for different Strike Price

Put Option

Put Premium Option when the Rate of Interest is changed

It can be seen that as the risk-free rate of interest is increased, the put premium decreases. This proves the theory of the impact of risk-free interest rate on the put option premium.

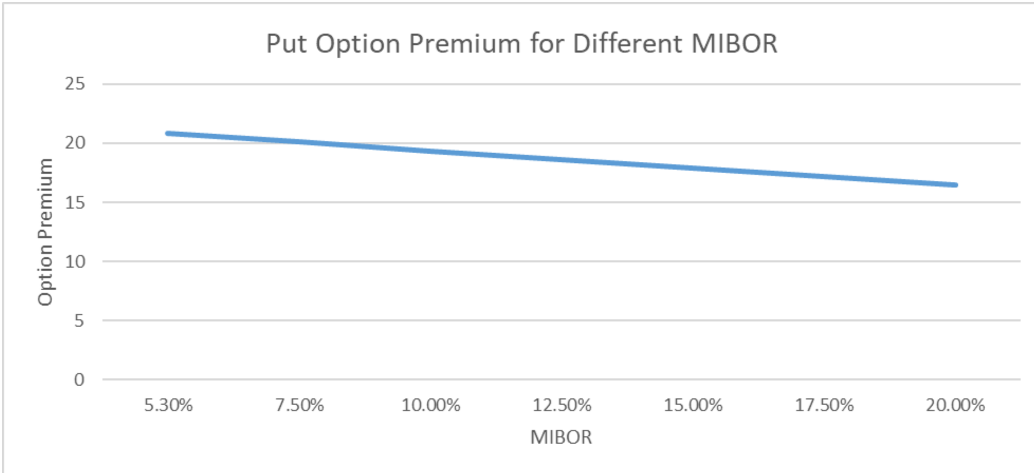


Figure 13 Change in Put Option Premium for different Rates of Interest

Put Premium Option when the Time Till Expiration is changed

It can be seen that as the time till expiration is increased, the put premium is also increased. This proves the theory of the impact of time till expiration on the put option premium.

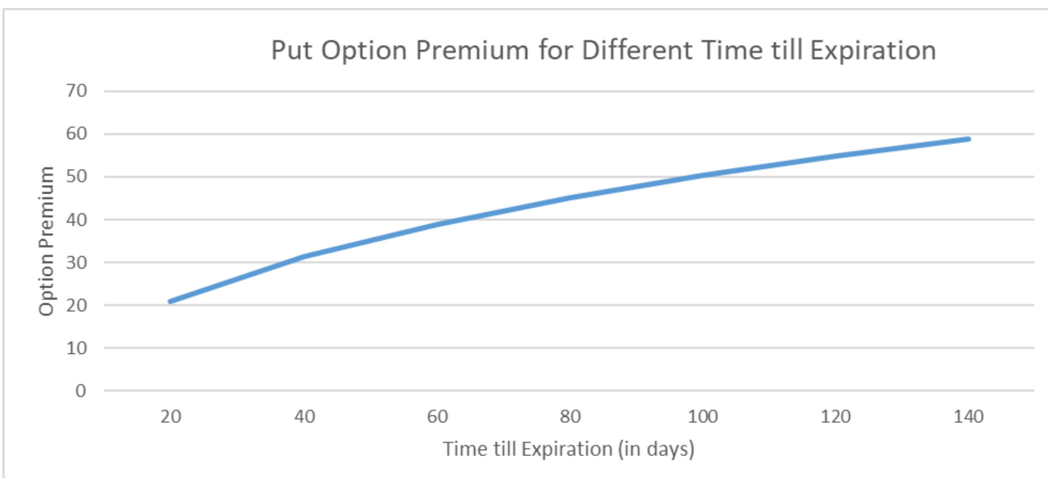


Figure 14 Change in Put Option Premium for different Time till Expiration

Put Premium Option when the Implied Volatility is changed

It can be seen that when the implied volatility is increased, the put option premium is increased. This proves the theory of the impact of implied volatility on the put option premium.

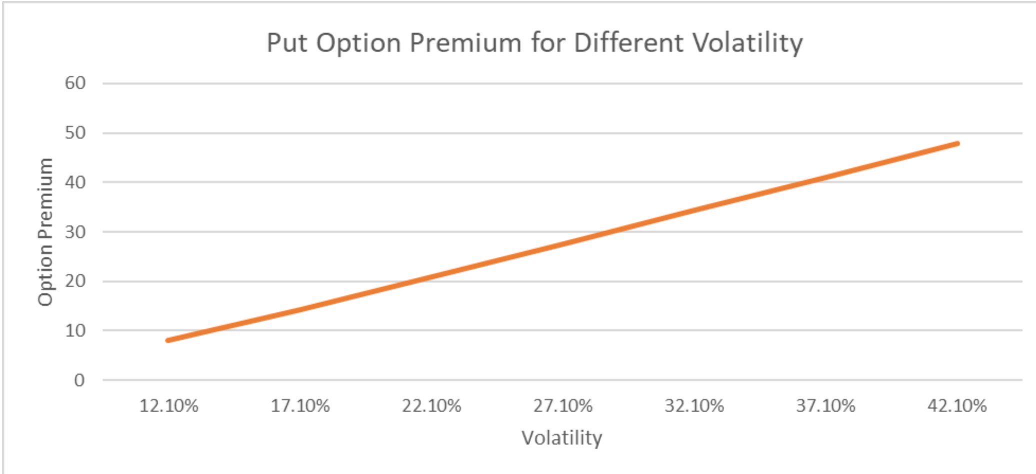


Figure 15 Change in Put Option Premium for different Volatility

Put Premium Option when the Stock/Spot Price is changed

It can be seen that when the spot price is increased, the put option premium is decreased. This proves the theory of the impact of spot price on the put option premium.

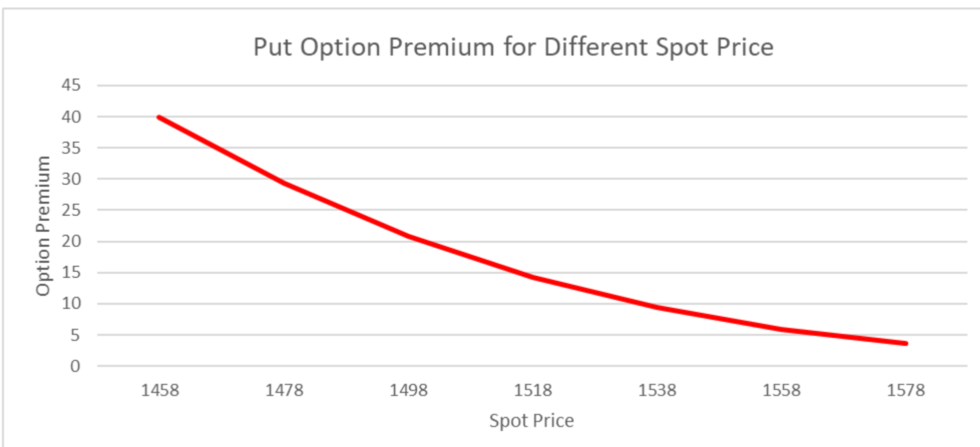


Figure 16 Change in Put Option Premium for different Spot Price

Put Option Premium when the stock price is changed

It can be seen that when the strike price is increased, the put option premium is decreased. This proves the theory of the impact of strike price on the put option premium.

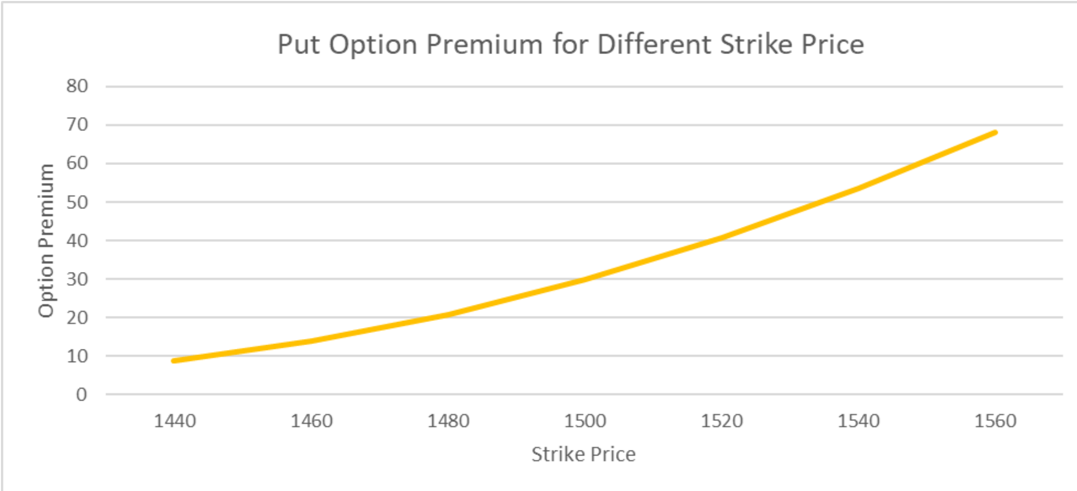


Figure 17 Change in Put Option Premium for different Stock Price

4.2 TEST FOR SIGNIFICANT DIFFERENCE BETWEEN FAIR PRICE AND ACTUAL PRICE

We have Calculated the Fair prices using Black Scholes Model in R-

For Calculation purpose we used:

Risk-Free Interest Rate – 6.26%

Volatility for NIFTY call options – 16.32%

Volatility for Bank NIFTY Put options – 25.31%

Calculation of Fair prices using R and Excel Files Showing Calculations:

https://drive.google.com/drive/folders/1IMgEoc6djiSw3a-N0yDfJnULCa1ZBi1U?usp=sharing

NIFTY Call Options (from 1st August,2022 to 31st August,2022)

DATE	EXPIRY DATE	OPTION TYPE	Actual Price	Fair price	Difference
30-Aug-22	15-Sep-22	CE	145.45	159.48	14.03
29-Aug-22	15-Sep-22	CE	43.80	49.51	5.71
26-Aug-22	15-Sep-22	CE	104.65	121.94	17.29
25-Aug-22	15-Sep-22	CE	112.55	117.92	5.37
24-Aug-22	15-Sep-22	CE	134.80	149.75	14.95
23-Aug-22	15-Sep-22	CE	136.20	147.47	11.27
22-Aug-22	15-Sep-22	CE	114.50	127.43	12.93
19-Aug-22	15-Sep-22	CE	218.85	242.81	23.96
18-Aug-22	15-Sep-22	CE	321.55	345.07	23.52
17-Aug-22	15-Sep-22	CE	317.90	345.88	27.98
16-Aug-22	15-Sep-22	CE	259.70	293.75	34.05
12-Aug-22	15-Sep-22	CE	223.20	263.57	40.37

11-Aug-22	**15-Sep-22**	**CE**	**231.40**	**253.57**	**22.17**
10-Aug-22	**15-Sep-22**	**CE**	**200.00**	**212.01**	**12.01**
8-Aug-22	**15-Sep-22**	**CE**	**191.20**	**219.82**	**28.62**
5-Aug-22	**15-Sep-22**	**CE**	**135.95**	**192.43**	**56.48**
4-Aug-22	**15-Sep-22**	**CE**	**135.95**	**192.52**	**56.57**
3-Aug-22	**15-Sep-22**	**CE**	**163.20**	**199.49**	**36.29**
2-Aug-22	**15-Sep-22**	**CE**	**160.00**	**190.86**	**30.86**
1-Aug-22	**15-Sep-22**	**CE**	**156.20**	**194.00**	**37.80**

Table 4.1 NIFTY Call Options Data

Mean of Differences = 25.612

Standard Deviation = 14.489

No. of observation (n) = 20

t = Mean/ (Sd/√n)

= 7.905527

P-value at (n-1) degree of freedom for t = 7.905527 is 0%

(calculated using P-value Calculator)

https://www.statology.org/t-score-p-value-calculator/

0% < 5%

So, we can reject null Hypothesis and can state that the difference between Fair and Actual Price is significant.

Bank NIFTY Put Options (from 18th August,2022 to 12th September,2022)

DATE	EXPIRY DATE	OPTION TYPE	Actual Price	Fair price	Difference
12-Sep-22	15-Sep-22	PE	142.00	145.07	3.07
9-Sep-22	15-Sep-22	PE	239.95	323.26	83.31
8-Sep-22	15-Sep-22	PE	325.00	441.12	116.12
7-Sep-22	15-Sep-22	PE	753.30	869.70	116.40
6-Sep-22	15-Sep-22	PE	680.25	776.05	95.80
5-Sep-22	15-Sep-22	PE	644.30	730.81	86.51
2-Sep-22	15-Sep-22	PE	896.50	1025.66	129.16
1-Sep-22	15-Sep-22	PE	929.10	1120.89	191.79
30-Aug-22	15-Sep-22	PE	810.00	1027.87	217.87
29-Aug-22	15-Sep-22	PE	1843.25	1885.19	41.94
26-Aug-22	15-Sep-22	PE	1100.00	1434.14	334.14
25-Aug-22	15-Sep-22	PE	1456.25	1474.20	17.95
24-Aug-22	15-Sep-22	PE	1600.00	1435.82	-164.18
23-Aug-22	15-Sep-22	PE	1600.00	1670.35	70.35

22-Aug-22	**15-Sep-22**	**PE**	**1600.00**	**1960.59**	**360.59**
19-Aug-22	**15-Sep-22**	**PE**	**1100.00**	**1546.33**	**446.33**
18-Aug-22	**15-Sep-22**	**PE**	**870.00**	**1188.40**	**318.40**

Table 4.2 Bank NIFTY Put Options

Mean of Differences = 145.032

Standard Deviation = 152.119

No. of observation (n) = 17

$t = \text{Mean}/(\text{Sd}/\sqrt{n})$

= 3.931010

P-value at (n-1) degree of freedom for t = 3.931010 is 0.119%

(calculated using P-value Calculator)

https://www.statology.org/t-score-p-value-calculator/

0.119% < 5%

So, we can reject null Hypothesis and can state that the difference between Fair and Actual Price is significant.

5. Conclusion/Outcomes

5.1 IMPACT OF CHANGE IN FACTORS ON OPTION PRICING

We checked for the relationship between different factors and their effect on the option pricing and compared it with the theoretical effect that it should have had. We can conclude that Black Scholes model behaves in the similar manner as it should have according to the theoretical change in intrinsic and extrinsic value. Hence, Black Scholes model can be seen as a fairly good estimate for the option pricing in terms of how it behaves when the factors change.

5.2 TEST FOR SIGNIFICANT DIFFERENCE BETWEEN FAIR PRICE AND ACTUAL PRICE

As, null hypothesis of both the call and put option is rejected. Thus, we can conclude fair value is not equal to the actual value. This can be because of our assumptions related to risk-free interest rate and volatility.

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